Angular Momentum Transport in MRI-driven Accretion Disks: Beyond the α-disk Models

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Image Credit: NASA

Angular Momentum Transport in Accretion Disks

$$\frac{\partial \bar{l}}{\partial t} + \bar{\nabla} \cdot (\bar{l}\bar{\mathrm{v}}) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r^2 \bar{T}_{r\phi}\right)$$

MHD Stress

$$\bar{T}_{r\phi} = \bar{R}_{r\phi} - \bar{M}_{r\phi}$$

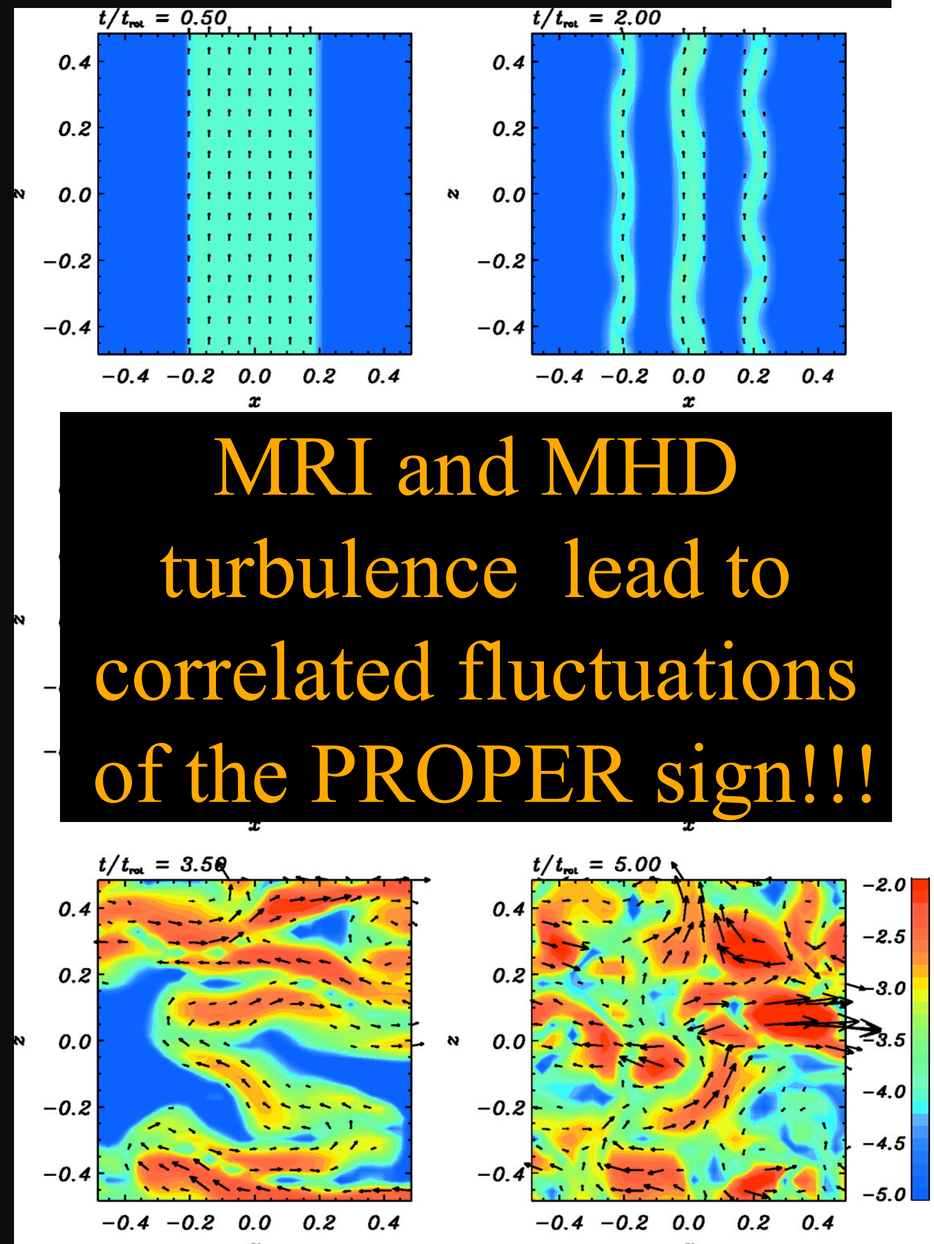
Reynolds Stress

$$\bar{R}_{r\phi} = \left\langle \rho \delta \mathrm{v}_r \delta \mathrm{v}_\phi \right\rangle$$

Maxwell Stress

$$\bar{M}_{r\phi} = \frac{\left\langle \delta B_r \delta B_\phi \right\rangle}{4\pi}$$

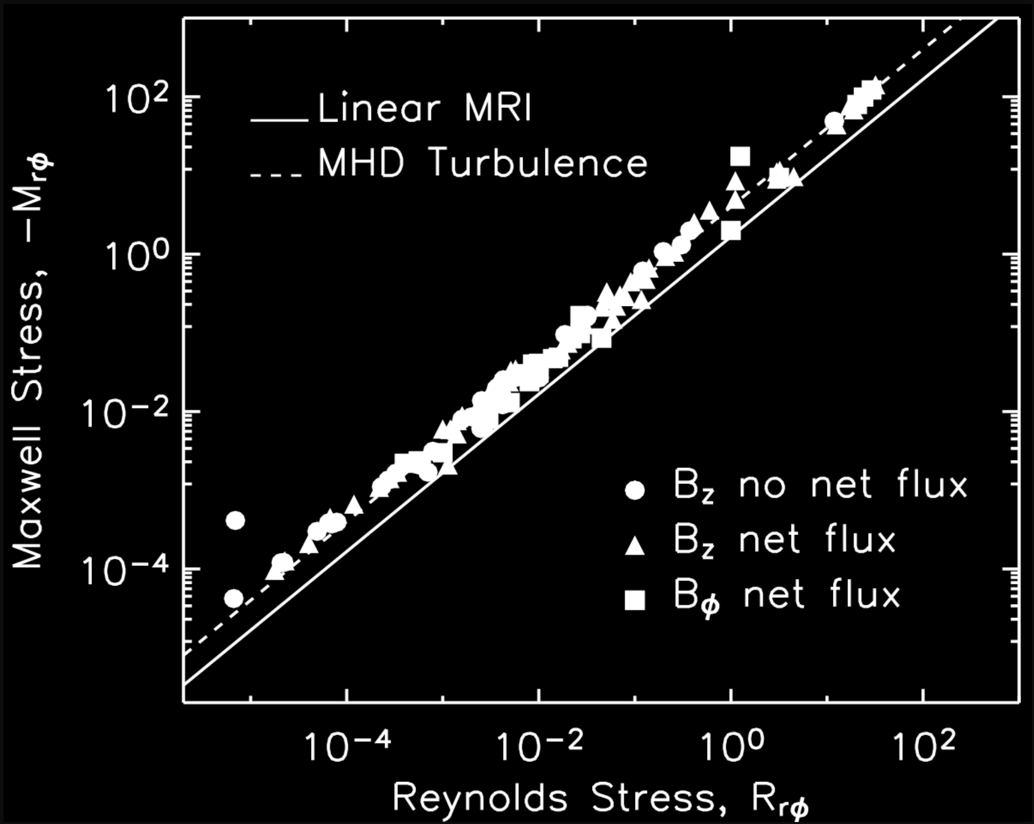
Sano et al. 2004

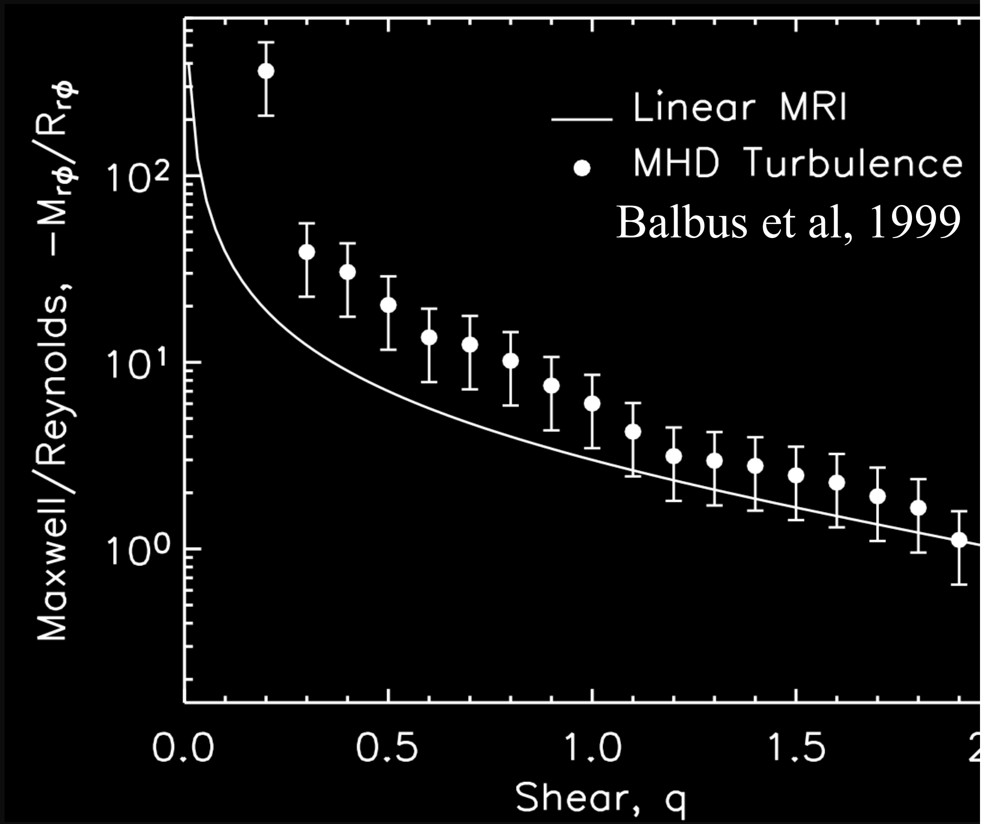


MRI and MHD turbulence lead to correlated fluctuations of the PROPER sign!!!

Linear MRI vs. MHD Turbulence

Pessah, Chan, & Psaltis, 2006a





The ratio between the Reynolds and the Maxwell stresses depends only on the local shear q = -dlog Ω/ dlog r

Stresses in MRI-driven Turbulence

❖ We need equations for the Reynolds and Maxwell stresses!

$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\bar{R}_{r\phi} = 2\bar{R}_{\phi\phi} - \frac{\kappa^2}{2}\bar{R}_{rr} - \bar{W}_{rr} + \bar{W}_{\phi\phi}$$

$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\bar{M}_{r\phi} = -q\bar{M}_{rr} + \bar{W}_{rr} - \bar{W}_{\phi\phi}$$

$$\overline{R}_{ik} = \langle \rho\delta v_i \delta v_k \rangle$$

$$\overline{M}_{ik} = \langle \delta B_i \delta B_k \rangle$$

❖ A new correlation appears naturally!

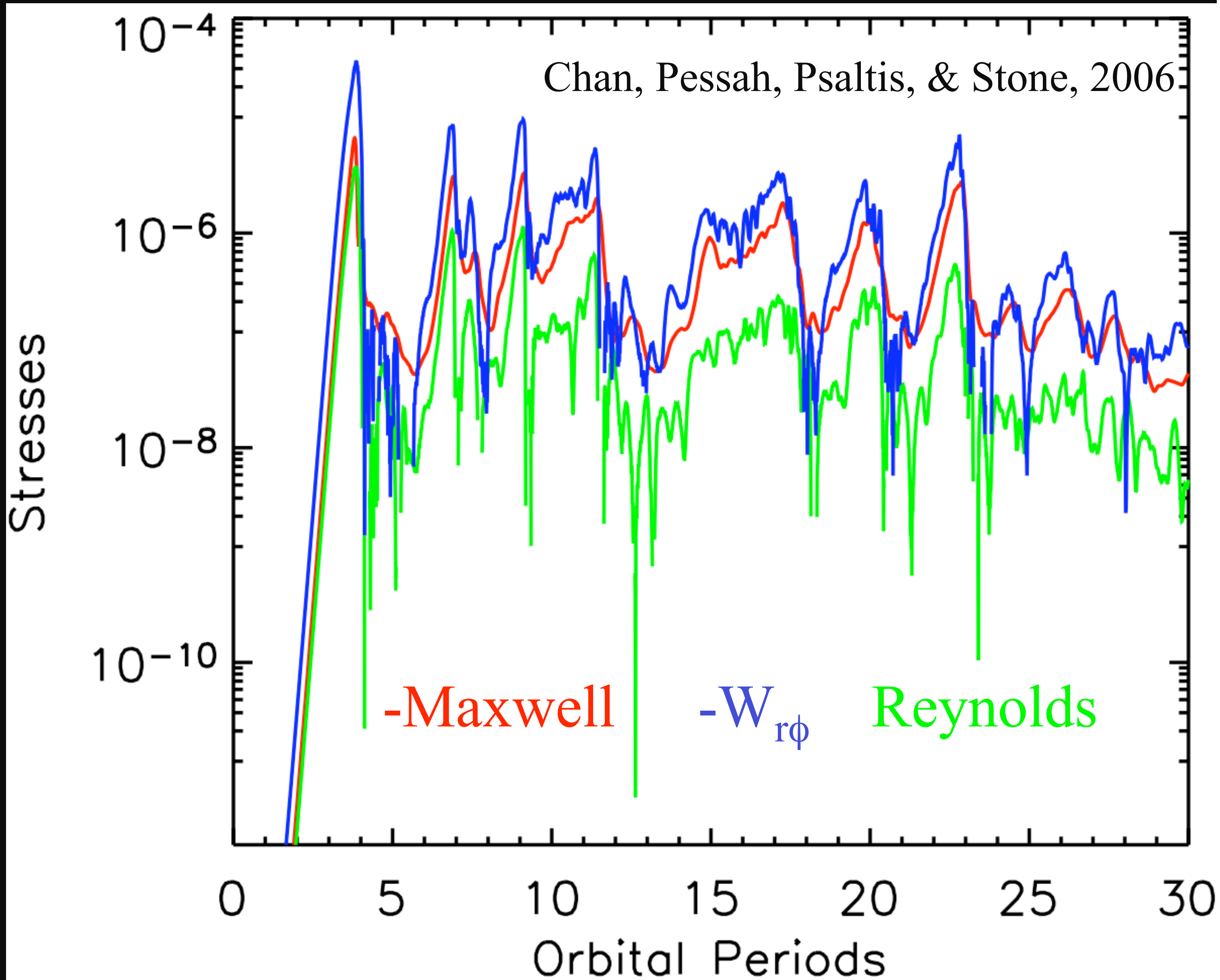
$$\overline{W}_{ik} = \langle \delta v_i \delta j_k \rangle$$

❖ W_{ij} drives the turbulence!!!

W in the Turbulent State

❖W tensor dynamically important!

❖We need equations for this new correlation!



MRI-driven stresses

$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\bar{R}_{r\phi} = 2\bar{R}_{\phi\phi} - \frac{\kappa^2}{2}\bar{R}_{rr} - \bar{W}_{rr} + \bar{W}_{\phi\phi} - \sqrt{\frac{\bar{M}}{\bar{M}_0}}\bar{R}_{r\phi}$$

$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\bar{W}_{rr} = q\bar{W}_{r\phi} + 2\bar{W}_{\phi r} + \zeta^2 k_{\max}^2(\bar{R}_{r\phi} - \bar{M}_{r\phi}) - \sqrt{\frac{\bar{M}}{\bar{M}_0}}\bar{W}_{rr}$$

$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\bar{W}_{\phi\phi} = -\frac{\kappa^2}{2}\bar{W}_{r\phi} - \zeta^2 k_{\max}^2(\bar{R}_{r\phi} - \bar{M}_{r\phi}) - \sqrt{\frac{\bar{M}}{\bar{M}_0}}\bar{W}_{\phi\phi}$$

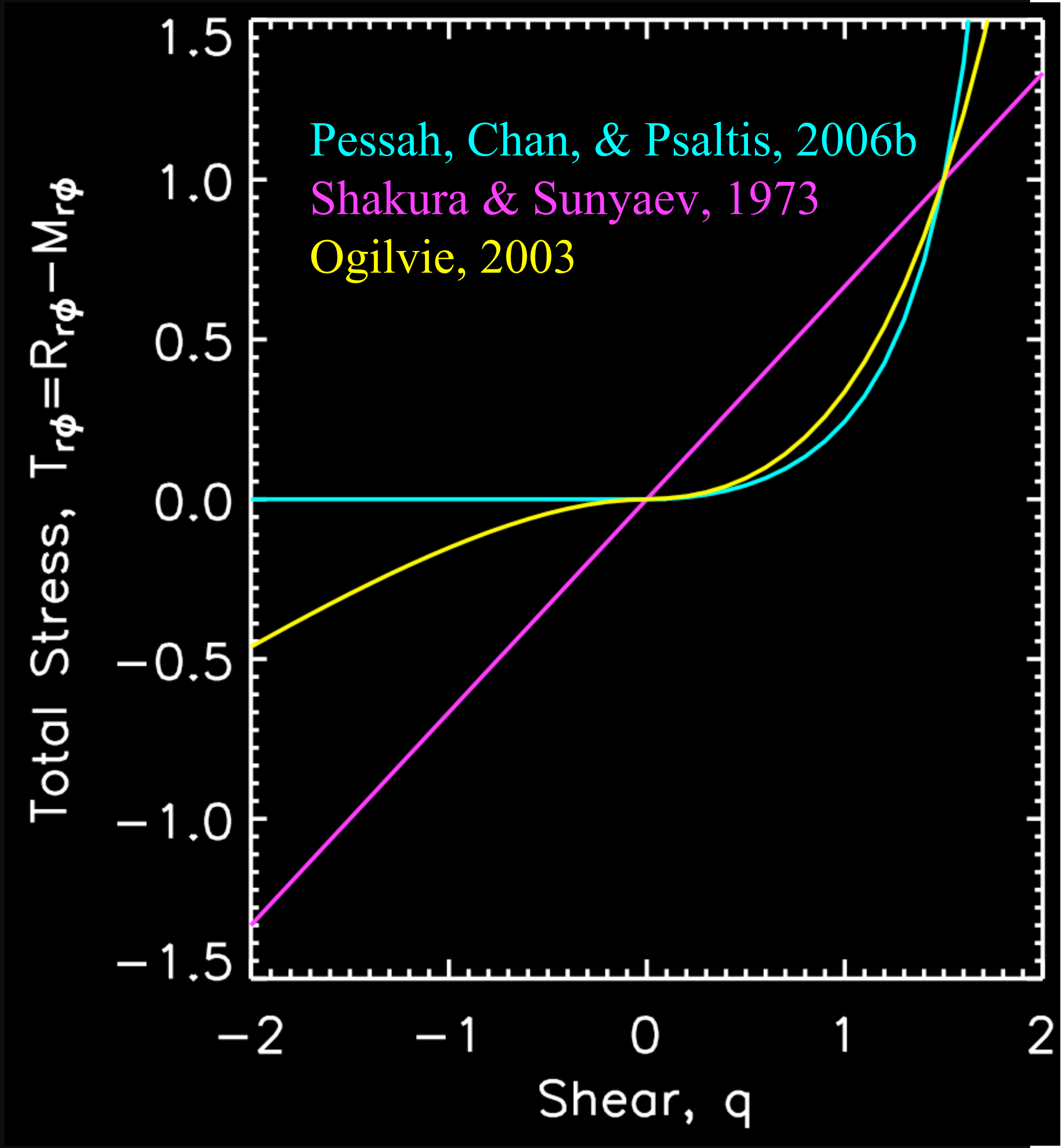
$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\bar{M}_{r\phi} = -q\bar{M}_{rr} + \bar{W}_{rr} - \bar{W}_{\phi\phi} - \sqrt{\frac{\bar{M}}{\bar{M}_0}}\bar{M}_{r\phi}$$

$$\bar{k}_{ij} = \zeta\, k_{\max}$$

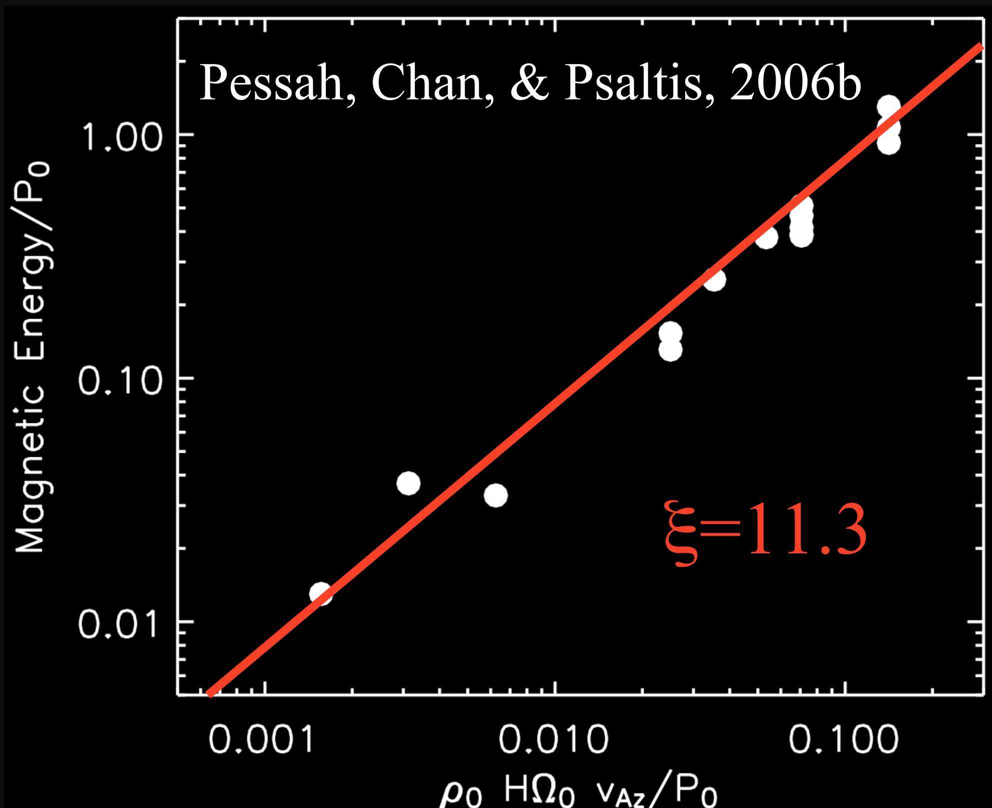
$$\bar{M}_0 = \xi\rho_0 H\Omega_0 \bar{v}_{Az}$$

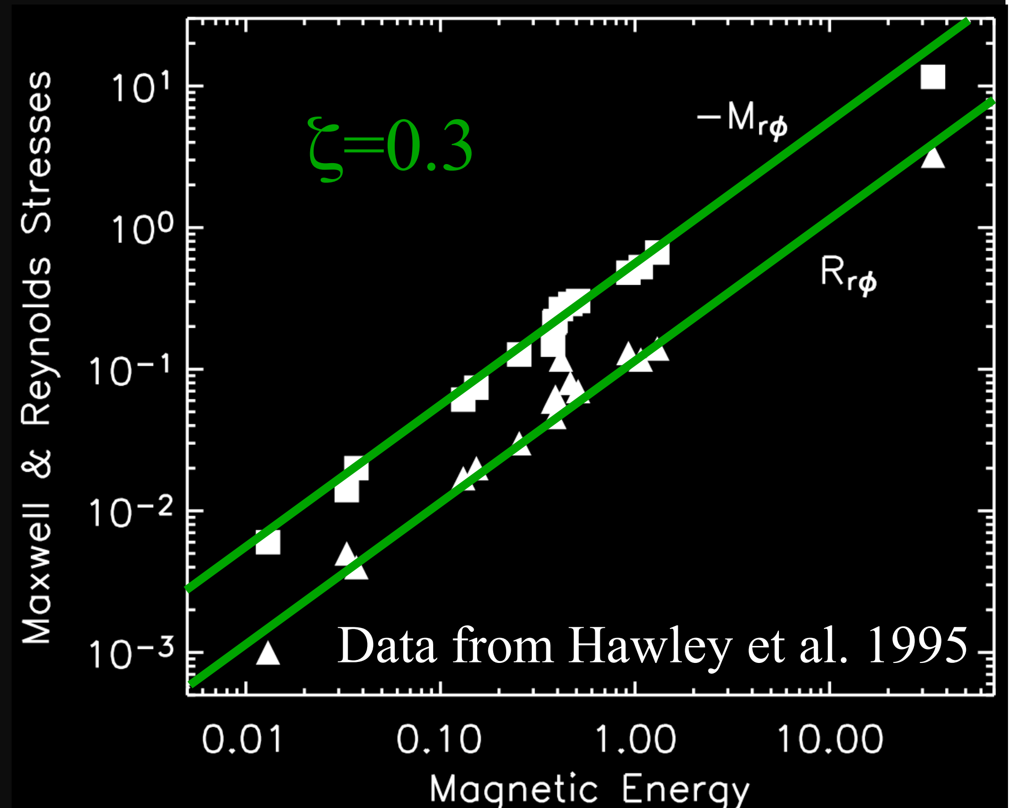
Properties of the Model

- Physically motivated
- Has only two model parameters
- Works when the disk is MRI-unstable
- Does not reduce to α-viscosity



Model Calibration Using Numerical Simulations





A model for angular momentum transport in turbulent MHD shearing flows that describes accurately the results of local numerical simulations.